

Année universitaire 2024/2025

# Mathématiques de l'Assurance de l'Economie et de la Finance - 2e année de Master

**Crédits ECTS : 60**

## LES OBJECTIFS DE LA FORMATION

Le parcours MASEF forme des mathématiciens probabilistes de haut niveau aux techniques mathématiques utilisées en gestion des risques et en économie. L'équilibre entre les cours fondamentaux et les cours à vocation plus appliquée assure aux étudiants une formation leur permettant selon leur choix d'options de s'orienter vers la recherche académique ou vers des postes à forte composante quantitative dans l'industrie. Les principaux débouchés sont les banques mais également les secteurs nécessitant des compétences rares en matière d'optimisation en environnement aléatoire et/ou incertain (industrie de l'énergie, enchères sur le web, etc.).

Les objectifs de la formation :

- Maîtriser les fondamentaux de l'optimisation, de la décision et de la couverture de risques en environnement incertain avec ou sans interactions.
- Maîtriser des outils numériques et statistiques, dont les techniques d'apprentissage automatique.
- Maîtriser des techniques de finance quantitative avancées.
- Acquérir du recul nécessaire à s'adapter aux évolutions, que ce soit dans le domaine de la recherche ou de l'industrie.

## PRÉ-REQUIS OBLIGATOIRES

- Titulaires d'un diplôme BAC+4 (240 crédits ECTS) ou équivalent à Dauphine, d'une université, d'une grande école ou d'un autre établissement de l'enseignement supérieur dans les domaines suivants : mathématiques pures, mathématiques Appliquées

## POURSUITE D'ÉTUDES

La majorité des étudiants trouve un poste à fort contenu quantitatif dans les banques, fonds d'investissement, brokers, compagnies d'assurance, sociétés de service en finance de marché (quant, structuration, trading, gestion quantitative de portefeuille).

La formation prépare naturellement, selon les options choisies, à une thèse de doctorat en finance mathématique, assurance, économie mathématique, mais également en probabilités numérique. Chaque année, deux à cinq étudiants poursuivent la spécialité par une thèse de doctorat, soit purement académique (thèse classique) soit en partenariat avec une cellule de recherche d'une entreprise privée (thèse Cifre)

Les thèses classiques sont en général effectuées au Ceremade ou au Crest et sont encadrées par l'école doctorale EDDIMO de Dauphine. Elles peuvent également être effectuées dans d'autres établissements (Université Paris 6, Paris 7, Ecole Polytechnique par exemple).

Débouchés : Analyste quantitatif, Chargé de modélisation, Consultant en Finance, Contrôleur des risques, Trading assistant, Valuation Analyst, ect.

## PROGRAMME DE LA FORMATION

- Semestre 1
  - UE Obligatoires
    - A review of probability theory foundations
    - Stochastic Calculus
    - Stochastic Control
    - Monte-Carlo and Finite Differences Methods with Applications to Finance
  - UE Optionnelles
    - Machine Learning in finance
    - Continuous Optimization
    - Valuation of financial assets by absence of arbitrage
    - Jump processes
    - Game theory, applications in economics and finance
    - Macro-économiques et gestion de portefeuille
    - Computational methods and MCMC
    - Term Structure and Commodity Derivatives Markets
    - Derivative products in finance and insurance
- Semestre 2
  - UE fondamentale
    - Conferences cycle: strategies and actors in portfolio management
  - Bloc 1 : Apprentissage pour l'économie et la finance
    - Rough volatility, microstructure and statistical modelling
    - Renforcement Learning
    - Python/Pytorch project
  - Bloc 2 : Finance et gestion des risques
    - Gestion globale des risques : VAR
    - Microstructure des marchés financiers
    - Contrôle stochastique et marchés de l'énergie
    - Modélisation stochastique des courbes de taux
  - Bloc 3 : Economie et jeux
    - Mean field games theory
    - Variational and transport problems in economics
    - Théorie géométrique du contrôle
    - Algèbre tropicale en optimisation et en jeux
    - Managing nature : the case of Fisheries

## DESCRIPTION DE CHAQUE ENSEIGNEMENT

### A review of probability theory foundations

ECTS : 0

#### Description du contenu de l'enseignement :

Outline :

1. Basics of measure theory and integration
2. Probability : random variables, independence
3. Convergence of random variables
4. Law of Large Numbers and Central Limit Theorem
5. Conditional expectations
6. Martingales in discrete time
7. Gaussian vectors
8. Brownian motion : definition, existence, first properties

**Compétence à acquérir :**

The aim of this class is to provide a quick review of the probability theory that is required to follow the 1st semester classes in MATH, MASEF and MASH.

Most of the content should already be familiar to students with a M1 in Mathematics.

---

## Algèbre tropicale en optimisation et en jeux

ECTS : 6

**Description du contenu de l'enseignement :**

Algèbre tropicale et structures de « caractéristique un » (semi-corps max-plus). Résolution de problèmes avec paiement moyen via des problèmes spectraux non-linéaires. Représentation des opérateurs de Shapley. Théorie de Perron-Frobenius non-linéaire, dynamiques monotones ou non-expansives. Existence du paiement moyen. Certificats de Collatz-Wielandt. Généralisation non-linéaire de l'algorithme de la puissance. Algorithmes d'itération sur les politiques. Résultats de complexité pour les jeux répétés.

**Compétence à acquérir :**

Ce cours présente un certain nombre d'outils et résultats récents, inspirés de la géométrie tropicale, relatifs aux problèmes de contrôle ou de jeux répétés, déterministes ou stochastiques, avec une attention particulière pour les problèmes de paiement moyen ou en temps long ainsi que pour les aspects combinatoires et algorithmiques. Certains résultats sont illustrés par des exemples issus d'applications (optimisation du référencement, optimisation de la croissance en dynamique de population).

---

## Computational methods and MCMC

ECTS : 6

**Description du contenu de l'enseignement :**

Motivations  
Monte-Carlo Methods  
Markov Chain Reminders  
The Metropolis-Hastings method  
The Gibbs Sampler  
Perfect sampling  
Sequential Monte-Carlo methods

**Compétence à acquérir :**

This course aims at presenting the basics and recent developments of simulation methods used in statistics and especially in Bayesian statistics. Methods of computation, maximization and high-dimensional integration have indeed become necessary to deal with the complex models envisaged in the user disciplines of statistics, such as econometrics, finance, genetics, ecology or epidemiology (among others!). The main innovation of the last ten years is the introduction of Markovian techniques for the approximation of probability laws (and the corresponding integrals). It thus forms the central part of the course, but we will also deal with particle systems and stochastic optimization methods such as simulated annealing.

---

## Conferences cycle: strategies and actors in portfolio management

ECTS : 2

---

## Continuous Optimization

ECTS : 6

**Description du contenu de l'enseignement :**

This course will review the mathematical foundations for Machine Learning, as well as the underlying algorithmic methods and showcases some modern applications of a broad range of optimization techniques.

Optimization is at the heart of most recent advances in machine learning. This includes of course most basic methods (linear regression, SVM and kernel methods). It is also the key for the recent explosion of deep learning which are state of the art approaches to solve supervised and unsupervised problems in imaging, vision and natural language processing.

This course will review the mathematical foundations, the underlying algorithmic methods and showcases some modern applications of a broad range of optimization techniques. The course will be composed of both classical lectures and numerical sessions in Python. The first part covers the basic methods of smooth optimization (gradient descent) and convex optimization

(optimality condition, constrained optimization, duality). The second part will features more advanced methods (non-smooth optimization, SDP programming, interior points and proximal methods). The last part will cover large scale methods (stochastic gradient descent), automatic differentiation (using modern python framework) and their application to neural network (shallow and deep nets).

**Compétence à acquérir :**

The objective of this course is to learn how to recognize, manipulate and solve a relatively large class of emerging convex problems in areas such as, for example, learning, finance or signal processing.

---

## Contrôle stochastique et marchés de l'énergie

ECTS : 6

**Compétence à acquérir :**

Energy markets are a natural field of applications for stochastic control modelling framework. Historical applications go from water management to the pricing of swing and demand-side contracts. With the deregulation of electricity and gas markets, new applications have raised the attention of financial economists. In particular, the question of the optimal investment in generation assets in the context of climate change and the questions linked to retail competition. These domains are conducive to the utilization of stochastic differential games. This course is intended to provide a short introduction to the physics of energy market and extensive applications taken for financial and economical research papers. For their evaluation, students are expected to realize a study of a research paper for which they will provide a critical analysis of their understanding of the model, together with the reproduction of the results of the paper.

---

## Derivative products in finance and insurance

ECTS : 6

**Description du contenu de l'enseignement :**

Participants will learn how financial institutions can build and structured products, how they value and hedge them, and what they are done for.

**Compétence à acquérir :**

The aim of this lecture is to train students in the practical evaluation of derivative products and the control of the associated risks. It also introduces them to the new hybrid structured products that have recently appeared in insurance.

---

## Game theory, applications in economics and finance

ECTS : 6

**Description du contenu de l'enseignement :**

A- Basics of game theory:

1. Zero-sum games: value, optimal strategies, saddle points, minmax theorem.
2. N-layers normal form games: equilibria in dominant strategies, Nash equilibria, dominated strategies, Nash's existence theorem.
3. Extensive form: backward induction, subgame perfection, theorem of Kuhn-Zermelo, behavior strategies and Kuhn's theorem.

B- Applications:

1. Repeated games and cooperation, folk theorems.
2. Zero-sum repeated games with incomplete information on one side (Aumann-Maschler's model). Splitting lemma, uniform value.
3. Zero-sum stochastic games: dynamic structure, Shapley operator, theorems of Bewley-Kohlberg and Mertens-Neyman.

**Compétence à acquérir :**

The first part deals with the basics of game theory, the second one with applications in economics and finance. There will only be time to study 2 of the 3 applications (to be decided).

---

## Gestion globale des risques : VAR

ECTS : 2

**Description du contenu de l'enseignement :**

Introduction. Modèles dynamiques pour les prix d'actifs financiers. Agrégation des risques, normalité, asymétrie, queues de distributions épaisses. La valeur risquée. Définition et méthodologies de calcul de la VaR (historiques, Monte Carlo, analytiques). Présentation de RiskMetrics de J.P. Morgan. Données, méthodologie, interprétations. Application. La cartographie de RiskMetrics, risque sur les instruments financiers comptants et produits dérivés. Estimation des matrices de variances-covariances, volatilités et corrélations.

**Compétence à acquérir :**

Analyse des modèles mathématiques du risque de marché, étude des méthodes de gestion globales du risque de marché lorsque les sources d'incertitude sont multiples.

**Bibliographie, lectures recommandées :**

- Jorion Philippe Value at Risk McGraw-Hill, 2006 - Longestae, J More, L Introduction to riskmetrics Morgan Guaranty trust Company, 1995 - Hull J, Risk management and Financial - Institutions 5th editions Pearson, 2018 - Hull J, Options, futures and other derivatives Prentice-Hall Pearson Education, 2017 - Portait R, Poncet P Finance de Marché Dalloz , 2014

---

## Jump processes

**ECTS :** 6

**Description du contenu de l'enseignement :**

Poisson process, compound Poisson process,  
Infinitely divisible distributions,  
Random measures of Poisson,  
Lévy process,  
Decomposition of Lévy-Khintchine,  
Itô's formula for Lévy processes,  
Stochastic differential equations driven by a Lévy process,  
Equivalence of measures, Doleans-Dade exponential, Girsanov's theorem  
Merton's Model  
Hawkes' Process

**Compétence à acquérir :**

This course aims to master the techniques of analysis and stochastic calculation specific to jump processes. It complements the "Stochastic Calculation" course which is limited to processes with continuous paths.

---

## Machine Learning in finance

**ECTS :** 6

**Description du contenu de l'enseignement :**

- Introduction to statistical learning: The Vapnik Chervonenkis dimension, PAC learning and the calibration versus prediction paradigm.
- Supervised learning: SVM, Mercer's theorem and the kernel trick, C-SVMs, mu-SVMs, a few words on SVMs for regressions.
- Unsupervised learning: Single class SVMs, clustering, anomaly detection, equivalence of different approaches via duality.
- Introduction to random forests and ensemble methods: bias variance tradeoff, bootstrap method
- A few words on neural networks: backpropagation, deep learning.
- Remarks on parsimony and penalisation : Ridge and Lasso regressions, dual interpretation of Lasso.

**Compétence à acquérir :**

Some Statistical Learning results are presented and applied to credit rating, anomalies detection and yield curves modeling. The principal notions are presented in the context of these case studies in finance.

**Mode de contrôle des connaissances :**

Final exam

**Bibliographie, lectures recommandées :**

[https://web.stanford.edu/~hastie/ISLR2/ISLRv2\\_website.pdf](https://web.stanford.edu/~hastie/ISLR2/ISLRv2_website.pdf)

---

## Macro-économiques et gestion de portefeuille

ECTS : 6

**Compétence à acquérir :**

Les gestionnaires de portefeuille ont besoin de posséder certaines connaissances macroéconomiques de base pour mieux fonder leurs décisions d'investissement. La valeur dite fondamentale des différents actifs financiers ne peut être analysée sans une prise en compte des évolutions macroéconomiques prévisibles à moyen et long terme. De plus, les performances de court terme des différentes classes d'actifs financiers dépendent cruciallement des indicateurs macro-économiques, notamment en matière d'inflation et de croissance. Ce cours présentera notamment les méthodes dominantes utilisées par les praticiens de marché (économistes de marché, gestionnaires...) pour analyser et anticiper les évolutions macro-économiques, ainsi que les inflexions de politique monétaire. Il a une vocation appliquée et vise à donner à de futurs gestionnaires une bonne connaissance des instruments pratiques de prévision macroéconomique ainsi que des indications sur la meilleure façon d'utiliser ces instruments pour améliorer la performance de leur gestion.

---

## Managing nature : the case of Fisheries

ECTS : 6

**Description du contenu de l'enseignement :**

The purpose of the course is to provide scientific insight into the way modern society interacts with its environment. Fisheries provide a good example. They have been exploited since the earliest times to feed human populations, but since the industrial revolution they have undergone a dramatic transformation, leading in some cases to collapse, and transformation of the oceanic ecosystem. The first part of the course will model fish populations, the effect of commercial fishing, and of regulations such as subsidies and quotas. In the second part, the course will investigate how to take into account, not only the needs of the present generation, but also the needs of future generations, so that fisheries management strikes a balance between profit and conservation.

**Program**

Part 1: bioeconomics (6 sessions of 1:30 hour)

**I. Introduction to the oceans**

1. Global warming, acidification, desoxygenation. Consequences on marine populations
2. The two sides of fisheries: catches and alimentation. North/South disequilibrium

**II. The Economics : Gordon-Schaefer model and beyond**

1. The model, Allee effect, MSY
2. Economics: open vs. restricted access, the role of interest rates
3. Managements instruments :
  - Subsidies and taxes
  - Quotas, transferable or not
  - Protected marine areas

**III. Ecosystem models**

1. Using ECOPATH and ECOSYM
2. Alternative models and complementarity : OSMOSE APECOSM ATLANTIS EWE viability : what are they used for ? What is the complementarity ?

Part 2: Beyond optimization (6 sessions of 1:30 hour)

**IV. The concept of optimization (1 session)**

1. Individuals: utility function, expectations, time preference
2. Groups: Condorcet paradox, Pareto optimum,
3. Groups: Nash equilibrium

**V. The economics of natural resources (1 session)**

1. The unitary model : Ramsey
2. Solving for optimality :
  - Finding the equilibrium
  - Writing the HJB equation
  - Solving the HJB equation
3. Non-renewable resources :
  - The Hotelling rule

-The Hubbert curve

#### VI. The economics of fisheries (1 session)

1. The Gordon-Schaefer model as a particular case of the Ramsey model
2. The tipping point

#### VII. Intergenerational equity part 1 (2 sessions)

1. The Chichilnisky criterion and time inconsistency
2. The intergenerational game and equilibrium Markov strategies
3. The HJB equation
4. Finding equilibrium strategies

#### VIII. Intergenerational equity part 2 (1 session)

1. The Sumaila-Walters criterion and time inconsistency
2. The HJB equation
3. Finding equilibrium strategies

---

## Mean field games theory

**ECTS** : 6

### Description du contenu de l'enseignement :

Stochastic Control cours (1rst semester) is a necessary prerequisite.

Mean field games is a new theory developed by Jean-Michel Lasry and Pierre-Louis Lions that is interested in the limit when the number of players tends towards infinity in stochastic differential games. This gives rise to new systems of partial differential equations coupling a Hamilton-Jacobi equation (backward) to a Fokker-Planck equation (forward). We will present in this course some results of existence, uniqueness and the connections with optimal control, mass transport and the notion of partial differential equations on the space of probability measures.

### Compétence à acquérir :

Mastering of the mean field games technics.

### Bibliographie, lectures recommandées :

Notes on the course: <https://www.ceremade.dauphine.fr/~cardaliaguet/Enseignement.html#ENSEIGNEMENT>

---

## Microstructure des marchés financiers

**ECTS** : 6

### Description du contenu de l'enseignement :

The field of market microstructure combines theoretical modeling, institutional knowledge, and empirical analysis to understand how prices result from the interactions of traders in financial markets. The course aims to acquaint students with (i) the canonical models in microstructure, and (ii) econometric models used to test the predictions of microstructure models.

Course structure:

1. Trading Mechanisms
2. Measuring Liquidity
3. Price Dynamics and Liquidity
4. Trade Size and Market Depth
5. Empirical Analysis

### Compétence à acquérir :

Master the concepts of financial markets microstructure

### Mode de contrôle des connaissances :

Evaluation: assignment and final exam

### Bibliographie, lectures recommandées :

Foucault, Thierry, Marco Pagano, and Ailsa Röell, Market Liquidity: Theory, Evidence, and Policy, Oxford University Press, 2013.

---

# Modélisation stochastique des courbes de taux

ECTS : 3

## Description du contenu de l'enseignement :

1. Quelques outils de calcul stochastique : rappels
2. Généralités sur les taux d'intérêt
3. Produits de taux classiques
4. Modèle LGM à un facteur
5. Modèle BGM (Brace, Gatarek et Musiela) / Jamishidian
6. Modèles à volatilité stochastique

## Compétence à acquérir :

Ce cours est consacré aux modèles de taux d'intérêts à temps continu. Au travers de nombreux exemples, on décrira leurs utilisations pour évaluer les produits dérivés sur taux d'intérêt.

---

# Monte-Carlo and Finite Differences Methods with Applications to Finance

ECTS : 6

## Description du contenu de l'enseignement :

Generalities on Monte-Carlo methods

1. Generalities on the convergence of moment estimators
2. Generators of uniform law
3. Simulation of other laws (rejection method, transformation, ...)
4. Low discrepancy sequences

Variance reduction

1. Antithetical control
2. Payoff regularization
3. Control Variable
4. Importance sampling

Process simulation and payoff discretization

1. Black-Scholes model
2. Discretisation of SDEs
3. Diffusion's bridges and applications to Asian, barrier and lookback options.

Calculation of sensitivities (greeks)

1. Finite differences
2. Greeks in the Black-Scholes model
3. Tangent process and Greeks
4. Malliavin calculus, Greeks, conditional expectations and pricing of American options

Calculation of conditional expectations and valuation of American options.

1. Nested Monte Carlo approach
2. Regression Methods (Tsitsiklis Van Roy, Longstaff Schwartz)
3. Rogers' Duality

Finite difference methods: the linear case

1. Construction of classical schemes (explicit, implicit, theta-scheme)
2. Conditions for stability and convergence

Finite difference methods: the non-linear case

1. Monotonous schemes: General conditions of stability and convergence
2. Examples of numerical schemes: variational problems, Hamilton-Jacobi-Bellman equations.

## Compétence à acquérir :

This course provides an in-depth presentation of the main techniques for the evaluating of options using Monte Carlo techniques.

---



## Python/Pytorch project

ECTS : 6

### Description du contenu de l'enseignement :

The Python and PyTorch languages are commonly used to build ML/IA algorithms. The classroom course is complemented by a practical application thesis in economics or finance (e.g. Deep hedging, rapid calculation of expected shortfall, optimal portfolio management, high-frequency trading, solving semi-linear equations of the second order, variance reduction, etc.).

### Compétence à acquérir :

- Mastery of Python and PyTorch. Ability to build an ML/IA algorithm.

---

## Renforcement Learning

ECTS : 6

---

## Rough volatility, microstructure and statistical modelling

ECTS : 6

### Description du contenu de l'enseignement :

Rough volatility models were introduced by Gatheral and Rosenbaum in 2014. This new paradigm in financial engineering has had a considerable success ever since. In particular, it sheds new light on the interplay between high frequency data (and the microstructural properties of prices) and the more traditional stochastic volatility diffusion models used for risk hedging at coarse scales. In this course, we will adopt the point of view of statistical modelling: how to build a price model consistent with data together with broad guiding financial principles (like e.g. absence of arbitrage) that has some robustness across time scales ? We will follow the theory of Gatheral and Rosenbaum from a statistical perspective and discuss the following topics: point processes (in particular Hawkes processes), scaling limits for semimartingales, maximum likelihood estimation, nonparametric estimation for stochastic processes, fractional Brownian motion, simulation of rough Heston models and a bit of stochastic control. It is not necessary to have a background in statistics (we will cover all the required tools) but it is mandatory to have some proficiency in stochastic calculus (continuous semimartingales, stochastic integration, Itô formula, Girsanov theorem and the basics of SDEs)

### Bibliographie, lectures recommandées :

Textbooks :

- Aït-Sahalia, Y. and Jacod, J. (2014). High-frequency financial econometrics. Princeton University Press.
- Ibragimov, I. A. and Has' Minskii, R. Z. (2013). Statistical estimation: asymptotic theory, volume 16. Springer Science & Business Media.
- Revuz, D., and Yor, M. (2013). Continuous martingales and Brownian motion (Vol. 293). Springer Science & Business Media.

Research papers :

- Bacry, E., Delattre, S., Hoffmann, M., and Muzy, J.-F. (2013b). Some limit theorems for Hawkes processes and application to financial statistics. *Stochastic Processes and their Applications*, 123(7):2475–2499.
- El Euch, O., Fukasawa, M., and Rosenbaum, M. (2018a). The microstructural foundations of leverage effect and rough volatility. *Finance and Stochastics*, 22(2):241–280.
- Gatheral, J., Jaisson, T., and Rosenbaum, M. (2018). Volatility is rough. *Quantitative Finance*, 18(6):933–949.
- Hawkes, A. G. (2018). Hawkes processes and their applications to finance: a review. *Quantitative Finance*, 18(2):193–198.
- Jaisson, T., Rosenbaum, M., et al. (2015). Limit theorems for nearly unstable Hawkes processes. *The Annals of Applied Probability*, 25(2):600–631.
- Jusselin, P. and Rosenbaum, M. (2020). No-arbitrage implies power-law market impact and rough volatility. *Mathematical Finance*, 30(4):1309–1336.

---

## Stochastic Calculus

ECTS : 6

### Description du contenu de l'enseignement :

The course consists of four parts, each occupying roughly 6 hours:

- Preliminaries (Gaussian processes, Brownian motion, martingales, local martingales, variation, quadratic variation)
- Stochastic integration (Isometry extension, Wiener integral, Ito integral, martingale property)
- Stochastic differentiation (Itô processes, Itô's Formula, Girsanov's Theorem)
- Stochastic differential equations (existence and uniqueness, Markov property, generator, connections with PDEs).

**Compétence à acquérir :**

This course is a practical introduction to the theory of stochastic calculus, with an emphasis on examples and applications rather than abstract subtleties. [Click here for more information](#)

**Mode de contrôle des connaissances :**

Final written exam, in class.

**Bibliographie, lectures recommandées :**

[Click here for more information](#)

---

## Stochastic Control

ECTS : 6

**Description du contenu de l'enseignement :**

Relationship between conditional expectations and parabolic linear PDEs.  
Formulation of standard stochastic control problems: dynamic programming principle.  
Hamilton-Jacobi-Bellman equation  
Verification approach  
Viscosity solutions (definitions, existence, comparison)  
Application to portfolio management, optimal shutdown and switching problems

Teacher : Bruno BOUCHARD

**Compétence à acquérir :**

PDEs and stochastic control problems naturally arise in risk control, option pricing, calibration, portfolio management, optimal book liquidation, etc. The aim of this course is to study the associated techniques, in particular to present the notion of viscosity solutions for PDEs.

---

## Term Structure and Commodity Derivatives Markets

ECTS : 6

**Description du contenu de l'enseignement :**

The term structure is defined as the relationship between the spot price and the futures prices of a derivative instrument, for any delivery date. It provides useful information for hedging, arbitrage, investment and evaluation: it indeed synthesizes the information available in the market and the operators' expectations concerning the future price of the underlying asset.

In many derivative markets, especially in interest rates and in commodity markets, the concept of term structure is very important, because the contract's maturity increases as the markets come to fruition. In the Eurodollar market, for example the maturities reach 10 years.

Chapter 1 presents a general introduction to derivatives today.

Chapter 2 examines the traditional theories of commodity prices and the explanation of the relationships between spot and futures prices. It proposes an empirical review of the results obtained through these frameworks and explains why these theories are still investigated today. It finally shows how to apply these theories to other assets: exchange rates and interest rates.

The traditional theories are however a bit limited when the whole term structure is considered. As a result, there is a need for a long-term extension of the analysis, which is the very subject of the Chapter 3. We first present a dynamic analysis of the term structure. Then the focus turns towards term structure models. The examples rely on the case commodity prices but can be extended to interest rates. Simulations highlight the influence of the assumptions concerning the stochastic process retained for the state variables and the number of state variables. We then explain the econometric method usually employed for the estimation of the parameters. In the presence of non-observable variables, there is a need for filtering techniques. We present the method of the Kalman filters. Finally, we study two main applications, i.e. dynamic hedging and investment valuation.

Chapter 4 is devoted to the study of structural models, ie micro-founded equilibrium models that also examine the interactions between the physical and the derivative markets. In this situation the spot price becomes endogenous. The interactions between prices are studied thanks to rational expectations equilibriums.

**Compétence à acquérir :**

At the end of this course, the students must have a broad knowledge about the term structures of derivative prices: the theories, the valuation methods, the econometric techniques, the empirical tests as well as the applications.

They will also be trained to use their knowledge on this topic in order to develop a critical view on recent research articles.

This course is mandatory for all students enrolled in the cursus PhD Qualifying Year. It is optional for all other students of the M2 104.

**Mode de contrôle des connaissances :**

20% Participation ; 80% Final exam

**Bibliographie, lectures recommandées :**

Main references:

- Danthine J.P., Donaldson J.B., Intermediate Financial Theory, 2d Ed., Elsevier, 2005.
- Hull J., Options, futures and other derivatives, 9th Ed.
- Kolb R.W. , Overdahl J.A. , Futures, options, and swaps, 5th Ed., Blackwell, 2007.
- Williams J., The economic function of futures markets, Cambridge University Press, 1986
- Wilmott P., Paul Wilmott on Quantitative Finance, 3-volume set, 2nd Ed., Wiley, 2006.

---

## Théorie géométrique du contrôle

ECTS : 6

---

### Valuation of financial assets by absence of arbitrage

ECTS : 6

**Description du contenu de l'enseignement :**

Course outline:

- I. Discrete time modelling
  - I.1. Financial assets
  - I.2. The No arbitrage condition and martingale measures (FTAP)
  - I.3. Pricing and hedging of European options; market completeness and 2nd FTAP
  - I.4. Pricing and hedging of American options (in a complete market)
- II. Continuous time modelling
  - II.1. Financial assets as Itô processes : general theory
  - II.2. Markovian models : PDE pricing, delta-hedging (European options, barrier options, American options)
  - II.3. Local volatility models and Dupire's formula
  - II.4. Stochastic volatility models : how to deal with market incompleteness; (semi-)static hedging; specific models and their properties

**Compétence à acquérir :**

The lecture starts with discrete time models which can be viewed as a proxy for continuous settings, and for which we present in detail the theory of arbitrage pricing. We then develop on the theory of continuous time models. We start with a general Itô-type framework and then specialize to different situations: Markovian models, local and stochastic volatility models. For each of them, we discuss the valuation and the hedging of different types of options : plain Vanilla and barrier options, American options, options on realized variance, etc. Finally, we present several specific volatility models (Heston, CEV, SABR,...) and discuss their specificities.

**Bibliographie, lectures recommandées :**

Bouchard B. et Chassagneux J.F., Fundamentals and advanced Techniques in derivatives hedging, Springer, 2016.  
Lamberton D. et B. Lapeyre, Introduction au calcul stochastique appliqué à la finance, Ellipses, Paris, 1999.

---

## Variational and transport problems in economics

ECTS : 6

**Description du contenu de l'enseignement :**

**Chapter 1: Convexity in the calculus of variations**

- separation theorems, Legendre transforms, subdifferentiability,
- convex duality by a general perturbation argument, special cases (Fenchel-Rockafellar, linear programming, zero sum games,

Lagrangian duality)

-calculus of variations: the role of convexity, relaxation, Euler-Lagrange equations

### **Chapter 2: The optimal transport problem of Monge and Kantorovich**

-The formulations of Monge and Kantorovich, examples and special cases (dimension one, the assignment problem, Birkhoff theorem), Kantorovich as a relaxation of Monge

-Kantorovich duality

-Twisted costs, existence of Monge solutions, Brenier's theorem, Monge-Ampère equation, OT proof of the isoperimetric inequality

-the distance cost case and its connection with minimal flows

### **Chapter 3: Dynamic optimal transport, Wasserstein spaces, gradient flows**

-Wasserstein spaces

-Benamou-Brenier formula and geodesics, displacement convexity

-gradient flows, a starter: the Fokker-Planck equation, general theory for lambda-convex functionals

### **Chapter 4: Computational OT and applications**

-Entropic OT, Sinkhorn algorithm and its convergence

-Matching problems, barycenters,

-Wasserstein distances as a loss, Wasserstein GANs

### **Compétence à acquérir :**

Mastering of variational and optimal transport methods used in economy.

---