Variational problems and optimal transport

ECTS : 6

Volume horaire : 24

Description du contenu de l'enseignement :

Chapter 1: Convexity in the calculus of variations

- separation theorems, Legendre transforms, subdifferentiability
- convex duality by a general perturbation argument, special cases (Fenchel-Rockafellar, linear programming, zero sum games, Lagrangian duality)
- calculus of variations: the role of convexity, relaxation, Euler-Lagrange equations

Chapter 2: The optimal transport problem of Monge and Kantorovich
- The formulations of Monge and Kantorovich, examples and special cases (dimension one, the assignment problem, Birkhoff theorem), Kantorovich as a relaxation of Monge
- Kantorovich duality
- Twisted costs, existence of Monge solutions, Brenier's theorem, Monge-Ampère equation, OT proof of the isoperimetric inequality
- the distance cost case and its connection with minimal flows

Chapter 3: Dynamic optimal transport, Wasserstein spaces, gradient flows
- Wasserstein spaces
- Benamou-Brenier formula and geodesics, displacement convexity
- gradient flows, a starter: the Fokker-Planck equation, general theory for lambda-convex functionals

Chapter 4: Computational OT and applications
- Entropic OT, Sinkhorn algorithm and its convergence
- Matching problems, barycenters,
- Wasserstein distances as a loss, Wasserstein GANs