Pathwise (rough) stochastic analysis

ECTS : 6

Volume horaire : 24

Description du contenu de l'enseignement :

The pathwise (or rough) approach to stochastic analysis consists in a series of analytic methods, developed in the last two decades, which are able to deal with the functions of low regularity which arise naturally when considering stochastic objects. The crucial step is typically to identify a natural metric space on which the "noise-to-solution" map is continuous.
These methods allow on the one hand for a new (and often more natural and robust) point of view on classical objects (such as the SDE defined by Itô integration), while also allowing to deal with objects which were previously out of reach (such as ODE driven by fractional Brownian motion, or singular stochastic PDE).
The class will consist in an overview of these various techniques, with an emphasis on the simple case of Lyons' rough path theory. It will be mostly self-contained (a basic knowledge of stochastic calculus will be helpful).

Outline

1. Sewing lemma and Young integration
2. Basics of rough path theory (Rough path spaces, Rough integration and differential equations)
3. Application to stochastic equations (Brownian motion as a rough path, extensions to Gaussian processes)
4. Stochastic sewing (and applications to regularization by noise of ODE)
5. Introduction to singular SPDE (regularity structures or paracontrolled distributions)

The course will be taught at ENS.

Bibliographie, lectures recommandées :

P. Friz and M. Hairer, *A course on rough paths*