

Dynamics of semi-linear wave equation

ECTS : 6

Volume horaire : 28

**Description du contenu de l'enseignement :**

The aim of this course is to present recent developments concerning the dynamics of non-linear wave equations.

In the first part of the course, I will present some classical properties of linear wave equations (cf. [3, Chapter 5]): representation of solutions, finite speed of propagation, asymptotic behavior, dispersion and Strichartz inequalities [7, 5].

The second part of the course concerns semi-linear wave equations. After a presentation of the basic properties of these equations (local existence and uniqueness of solutions, conservation laws, transformations cf. e.g. [5, 6]), I'll give several examples of dynamics: scattering to a linear solution, self-similar behavior and solitary waves. I will also give results on the classification of the dynamics for the energy critical wave equation following [2, 4], and some elements of proofs, including the profile decomposition introduced by Bahouri and Gérard [1].

The prerequisites are the basics of classical real and harmonic analysis. This course can be seen as a continuation of the fundamental courses *Introduction to Nonlinear Partial Differential Equations* and *Introduction to Evolutionary Partial Differential Equations*, but can also be taken independently of these two courses.

This course will be taught at ENS.

**Bibliographie, lectures recommandées :**

1. Bahouri, H., and Gérard, P. High frequency approximation of solutions to critical nonlinear wave equations. *Amer. J. Math.* 121, 1 (1999), 131–175.
2. Duyckaerts, T., Kenig, C., and Merle, F. Classification of radial solutions of the focusing, energy-critical wave equation. *Camb. J. Math.* 1, 1 (2013), 75–144.
3. Folland, G. B. *Introduction to partial differential equations.*, 2nd ed. ed. Princeton, NJ: Princeton University Press, 1995.
4. Kenig, C. E. *Lectures on the energy critical nonlinear wave equation*, vol. 122 of CBMS Reg. Conf. Ser. Math. Providence, RI: American Mathematical Society (AMS), 2015.
5. Sogge, C. D. *Lectures on nonlinear wave equations*. Monographs in Analysis, II. International Press, Boston, MA, 1995.
6. Strauss, W. A. *Nonlinear wave equations*, vol. 73 of CBMS Regional Conference Series in Mathematics. Published for the Conference Board of the Mathematical Sciences, Washington, DC, 1989.
7. Tao, T. *Nonlinear dispersive equations*, vol. 106 of CBMS Regional Conference Series in Mathematics. Published for the Conference Board of the Mathematical Sciences, Washington, DC, 2006. Local and global analysis.

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