Introduction to evolution PDEs

ECTS : 6

Volume horaire : 37.5

Description du contenu de l'enseignement :

In a first part, we will present several results about the well-posedness issue for evolution PDE.

- Parabolic equation. Existence of solutions for parabolic equations by the mean of the variational approach and the existence theorem of J.-L. Lions.
- Transport equation. Existence of solutions by the mean of the characterics method and renormalization theory of DiPerna-Lions. Uniqueness of solutions thanks to Gronwall argument and duality argument.
- Evolution equation and semigroup. Linear evolution equation, semigroup and generator. Duhamel formula and mild solution. Duality argument and the well-posedness issue. Semigroup Hille-Yosida-Lumer-Phillips' existence theory.

In a second part, we will mainly consider the long term asymptotic issue.

- More about the heat equation. Smoothing effect thanks to Nash argument. Rescaled (self-similar) variables and Fokker-Planck equation. Poincaré inequality and long time asymptotic (with rate) in L2 Fisher information, log Sobolev inequality and long time convergence to the equilibrium (with rate) in L1.
- Entropy and applications. Dynamical system, equilibrium and entropy methods. Self-adjoint operator with compact resolvent. A Krein-Rutman theorem for conservative operator. Relative entropy for linear and positive PDE. Application to a general Fokker-Planck equation. Weighted L2 inequality for the scattering equation.
- Markov semigroups and the Harris-Meyn-Tweedie theory.

In a last part, we will investigate how the different tools we have introduced before can be useful when considering a nonlinear evolution problem.

- The parabolic-elliptic Keller-Segel equation. Existence, mass conservation and blow up. Uniqueness. Self-similarity and long time behavior.