Numerical methods for deterministic and stochastic problems

ECTS : 6

Volume horaire : 45

Description du contenu de l'enseignement :

This course is an introduction to methods for the numerical solution of deterministic and stochastic differential equations and numerical aspects of machine learning. It consists of three distincts parts and includes implementations using Python, FreeFEM++ and Keras/Tensorflow.

Part 1: numerical methods for deterministic partial differential equations

- finite difference methods
- finite element methods
- spectral methods
- review of numerical methods for ordinary differential equations

Part 2: Monte Carlo methods for particle transport

- Monte Carlo integration
- convergence and variance reduction
- transport equations starting from probability measures: examples and numerical methods including particle methods

Part 3: machine learning and numerical statistics

- high-dimensional statistics and machine learning
- stochastic optimization : SGD, Adam, RMSProp, etc.
- neural networks: architecture, generative paradigms (VAE, GANs, "stable diffusion")

Bibliographie, lectures recommandées :

Part 1

- Randall J. LeVeque, "Finite Difference Methods for Ordinary and Partial Differential Equations: Steady-State and Time-dependent Problems", SIAM (2007)
- Alexandre Ern, Jean-Luc Guermond, "Theory and Practice of Finite Elements", Springer (2004)
- Jie Shen, Tao Tang, Li-Lian Wang, "Spectral Methods. Algorithms, Analysis and Applications", Springer (2011)

Part 2

- C. Graham, D. Talay, "Stochastic Simulation and Monte Carlo Methods", Springer (2013)
- B. Lapeyre, E. Pardoux, R. Sentis, "Introduction to Monte-Carlo Methods for Transport and Diffusion Equations", OUP Oxford (2003)

Part 3

- Ian Goodfellow, Yoshua Bengio, Aaron Courville, "Deep Learning", The MIT Press (2016)
- Alain Berlinet, Christine Thomas-Agnan "Reproducing Kernel Hilbert Spaces in Probability and Statistics", Springer (2011)

See also G. Turinici's web site.