Limit theorems and large deviations

ECTS : 6

Volume horaire : 30

Description du contenu de l'enseignement :

The first part of the course (5*3 hours) is devoted to the study of convergence of probability measures on general (that is not necessarily R or Rn) metric spaces or, equivalently, to the convergence in law of random variables taking values in general metric spaces. If this study has its own interest it is also useful to prove convergence of sequences of random objects in various random models that appear in probability theory. The main example we have to keep in mind is Donsker theorem that states that the path of a simple random walk on Z converges after proper renormalization to a brownian motion. We will start this course with some properties of probability measures on metric spaces and in particular on C([0, 1]), the space of real continuous function on [0, 1]. We will then study convergence of probability measures, having for aim Prohorov theorem that provides a useful characterization of relative compatctness via tightness. Finally we will gather everything to study convergence in law on C([0, 1]) and prove Donsker therorem. If there is still time we will consider other examples of applicatioin. The main reference for this first part of the course is Convergence of probability measures, P. Billingsley (second edition).

The second part of the course will deal with the theory of large deviations. This theory is concerned with the exponential decay of large fluctuations in random systems. We will try to focus evenly on establishing rigorours results and on discussing applications. First, we will introduce the basic notions and theorems: the large deviation principle, Kramer theorem for independent variables, as well as GŁrtner-Ellis and Sanovs theorems. Next, we will see some applications of the formalism. The examples are mainly inspired by equilibrium statistical physics and thermodynamics. They include the equivalence of ensembles, the interpretation of thermodynamical potentials as large deviation functionals, and phase transitions in the mean-field Curie-Weiss model. In a third part, we will develop large deviation principles for Markovian dynamical processes. If times allows, we will present some applications of these results in a last part of the course. There is no explicit prerequisite to follow the classes but students should be well acquainted with probability theory.