Derivative Pricing and Stochastic calculus II (prerequisite: finance in continuous time)

ECTS : 6

Description du contenu de l'enseignement :

The aim of this lecture is to present the theory of derivative asset pricing as well as the main models and techniques used in practice. The lecture starts with discrete time models which can be viewed as a proxy for continuous settings. We then develop on the theory of continuous time models. We start with a general Itô-type framework and then specialize to different situations: Markovian models, constant volatility models, local and stochastic volatility models. For each of them, we discuss their calibration, and the valuation and the hedging of different types of options (plain Vanilla and barrier options, American options, options on realized variance,...).

Course outline:

I. Discrete time modelling
I.1. Financial assets I.2. The absence of arbitrage I.3. Pricing and hedging of European options I.4. Pricing and hedging of American options

II. Continuous time modelling
II.1. Financial assets as Itô processes II.2. The Black-Scholes model II.3. Markovian models in complete markets II.4. Local volatility models II.5. Stochastic volatility models

- General setting
- Tree markets
- Risk-neutral measures
- Fundamental theorem of asset pricing
- The super-hedging problem
- The complete market case : example of the CRR model
- Approximate hedging in incomplete markets
- Examples: binomial and trinomial tree markets
- The Itô process framework
- Discussion of the Absence of arbitrage opportunity
- Complete and incomplete markets
- The general pricing and hedging principle for European and American claims
- Characterization of complete Black Scholes markets
- Explicit formulas : European call option (Black-Scholes formula), barrier option (reflection principle)
- PDE valuation (plain vanilla, barrier, Asian, American options
- Greeks and hedging
- Tracking error and convexity
- Dupire’s formula and calibration to the volatility surface
- Super hedging prices
- Completion of the market with options : general principle, Approximate static hedging: example of the variance swap hedging problem
- Specific models : CEV, Heston, SABR,...

Compétence à acquérir :

The lecture starts with discrete time models which can be viewed as a proxy for continuous settings, and for which we present in detail the theory of arbitrage pricing. We then develop on the theory of continuous time models. We start with a general Itô-type framework and then specialize to different situations: Markovian models, local and stochastic volatility models. For each of them, we discuss the valuation and the hedging of different types of options : plain Vanilla and barrier options, American options, options on realized variance, etc. Finally, we present several specific volatility models (Heston, CEV, SABR,...) and discuss their specificities.

Mode de contrôle des connaissances :

Final exam

Bibliographie, lectures recommandées :

Bouchard B. et Chassagneux J.F., Fundamentals and advanced Techniques in derivatives hedging, Springer, 2016.
Lamberton D. et B. Lapeyre, Introduction au calcul stochastique appliqué à la finance, Ellipses, Paris, 1999.