Année universitaire 2025/2026

Mathématiques Appliquées et Théoriques - 2e année de Master

Responsables pédagogiques :

- ERIC **SERE** - https://dauphine.psl.eu/recherche/cvtheque/sere-eric
- CRISTINA **TONINELLI** - https://dauphine.psl.eu/recherche/cvtheque/toninelli-cristina

Crédits ECTS : 60

LES OBJECTIFS DE LA FORMATION

Ce parcours est une restructuration de l'actuel parcours de M2 Analyse et Probabilités opéré par l'Université Paris Dauphine - PSL, dont un tiers des cours sont déjà proposés par les autres établissements de PSL. Le parcours Mathématiques Appliquées et Théoriques est une formation de pointe qui prépare les étudiants à un doctorat en mathématiques en leur offrant une formation théorique solide dans divers champs de mathématiques.

Les objectifs de la formation :

- Etre en capacité de modélisation et d'analyse
- Avoir un socle de connaissances solides en mathématiques appliquées et théoriques
- Aptitude à la recherche et à la recherche et développement
- Acquérir dans un large champ des mathématiques, et en particulier dans les domaines suivants : analyse des EDP, contrôle et analyse numérique probabilités et physique statistique géométrie et systèmes dynamiques.

ADMISSIONS

Titulaires d'un diplôme BAC+4 (240 crédits ECTS) ou équivalent à Dauphine, d'une université ou d'un autre établissement de l'enseignement supérieur dans le domaine des mathématiques appliquées

POURSUITE D'ÉTUDES

Doctorat de mathématiques appliquées et théorique.
Ingénieur recherche et développement.

PROGRAMME DE LA FORMATION

- Semestre 3 - 42 ECTS
  - Cours introductifs
    - A review of functional analysis tools for PDEs
    - A review of probability theory foundations
    - A review of differential calculus for ODEs and PDEs
  - Cours fondamentaux
    - Continuous-time Markov processes
    - Introduction to dynamical systems
    - Introduction to evolution PDEs
    - Introduction to non linear elliptic PDEs
    - Limit theorems and large deviations
    - Numerical methods for deterministic and stochastic problems
    - Stochastic calculus
  - Cours spécialisés
    - An introduction to Hyperbolic Systems of Conservation Laws
    - Bayesian statistics

        - Continuous optimization
        - Course of PDE and applications
        - Determinantal processes, random matrices and hyperuniformity
        - Differential geometry and gauge theory
        - Dimension reduction and manifold learning
        - Dynamics of semi-linear wave equation
        - Dynamics of gravitational systems with a large number of particles
        - Entropy methods, functional inequalities and applications
        - Gravitation classique et Mécanique Hamiltonienne
        - High-dimensional statistics
        - Integrable probability and the KPZ universality class
        - Introduction to control theory
        - Introduction to generative modeling with flows and diffusions (ENS)
        - Introduction to statistical mechanics
        - Lie Groups, Lie algebras and representions
        - Mean field games theory
        - Random geometric models
        - Random walks and random media
        - Rough paths and SDE
        - Spectral theory and variational methods
        - Stochastic control
        - Variational and geodesic methods for Image Analysis
        - Variational problems and optimal transport
- Semestre 4 - 18 ECTS
    - Bloc mémoire
        - Mémoire de recherche

DESCRIPTION DE CHAQUE ENSEIGNEMENT

SEMESTRE 3 - 42 ECTS

Cours introductifs

A review of functional analysis tools for PDEs

Langue du cours : Anglais

Volume horaire : 15

Description du contenu de l'enseignement :

- Lp spaces, Sobolev spaces
- Distributions, Fourier transform, Laplace, heat and Schrödinger equations in the whole space
- Self-adjoint compact operators

Compétences à acquérir :

Basic techniques in Functional analysis

A review of probability theory foundations

Langue du cours : Anglais

Volume horaire : 15

Description du contenu de l'enseignement :

- Measure theory and integration
- Random variables, independence, inequalities
- Convergence of random variables, limit theorems
- Conditioning
- Stochastic processes, stopping times, martingales
- Gaussian vectors
- Brownian motion

Compétences à acquérir :

- Measure theory and integration
- Random variables, independence, inequalities
- Convergence of random variables, limit theorems
- Conditioning
- Stochastic processes, stopping times, martingales
- Gaussian vectors
- Brownian motion

A review of differential calculus for ODEs and PDEs

Langue du cours : Anglais

Volume horaire : 15

Description du contenu de l'enseignement :

We will revise the main notions and theorems from differential calculus (implicit function theorem, inverse function theorem, Brouwer theorem...), as well as main facts about ODE and results about linear and nonlinear stability and smooth dependance by perturbations.

Compétences à acquérir :

Cours fondamentaux

Continuous-time Markov processes

ECTS : 6

Enseignant responsable : JUSTIN **SALEZ** (https://dauphine.psl.eu/recherche/cvtheque/salez-justin)

Langue du cours : Anglais

Volume horaire : 45

Introduction to dynamical systems

ECTS : 6

Enseignants : JACQUES **FEJOZ**, ANNA **FLORIO**
https://dauphine.psl.eu/recherche/cvtheque/fejoz-jacques
https://dauphine.psl.eu/recherche/cvtheque/florio-anna

Langue du cours : Anglais

Volume horaire : 30

Description du contenu de l'enseignement :

1. Examples of dynamical systems in discrete and continuous time (circle rotation, shift, hyperbolic dynamical system, horseshoe, flow, section and suspension, attractor)
2. Topological dynamics, circle homeomorphisms and Poincaré classification, hyperbolic dynamics (geodesic flow, horocyclic flow)

Compétences à acquérir :

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Bibliographie, lectures recommandées :

- V.I. Arnold, Ordinary differential equations (contains prerequisite matters)
- V.I. Arnold, Geometric methods in the theory or ordinary differential equations (further reading)
- M. Brin and G. Stuck, Introduction to dynamical systems (great introduction to the field)

Introduction to evolution PDEs

ECTS : 6

Enseignant responsable : STEPHANE **MISCHLER** (https://dauphine.psl.eu/recherche/cvtheque/mischler-stephane)

Langue du cours : Anglais

Volume horaire : 37.5

Description du contenu de l'enseignement :

In a first part, we will present several results about the well-posedness issue for evolution PDE.

- Parabolic equation. Existence of solutions for parabolic equations by the mean of the variational approach and the existence theorem of J.-L. Lions.
- Transport equation. Existence of solutions by the mean of the characterics method and renormalization theory of DiPerna-Lions. Uniqueness of solutions thanks to Gronwall argument and duality argument.
- Evolution equation and semigroup. Linear evolution equation, semigroup and generator. Duhamel formula and mild solution. Duality argument and the well-posedness issue. Semigroup Hille-Yosida-Lumer-Phillips' existence theory.

In a second part, we will mainly consider the long term asymptotic issue.

- More about the heat equation. Smoothing effect thanks to Nash argument. Rescaled (self-similar) variables and Fokker-Planck equation. Poincaré inequality and long time asymptotic (with rate) in L2 Fisher information, log Sobolev inequality and long time convergence to the equilibrium (with rate) in L1.
- Entropy and applications. Dynamical system, equilibrium and entropy methods. Self-adjoint operator with compact resolvent. A Krein-Rutman theorem for conservative operator. Relative entropy for linear and positive PDE. Application to a general Fokker-Planck equation. Weighted L2 inequality for the scattering equation.
- Markov semigroups and the Harris-Meyn-Tweedie theory.

In a last part, we will investigate how the different tools we have introduced before can be useful when considering a nonlinear evolution problem.

- The parabolic-elliptic Keller-Segel equation. Existence, mass conservation and blow up. Uniqueness. Self-similarity and long time behavior.

Introduction to non linear elliptic PDEs

ECTS : 6

Enseignant responsable : ERIC **SERE** (https://dauphine.psl.eu/recherche/cvtheque/sere-eric)

Langue du cours : Anglais

Volume horaire : 37.5

Description du contenu de l'enseignement :

- Existence of weak solutions of linear and nonlinear elliptic PDEs by variational methods
- Regularity of weak solutions to linear and nonlinear elliptic PDEs
- Maximum principles and applications
- Brouwer degree, Leray-Schauder degree, fixed-point theorems
- Local and global bifurcation theory applied to nonlinear elliptic PDEs

Compétences à acquérir :

Nonlinear elliptic PDEs

Bibliographie, lectures recommandées :

- L.C. Evans, Partial Differential equations (Graduate Studies in Mathematics 19, AMS).
- L. Nirenberg, Topics in Nonlinear Functional Analysis (Courant Lecture Notes Series 6, AMS).

Limit theorems and large deviations

ECTS : 6

Enseignants : JULIEN **POISAT**, FRANCOIS **SIMENHAUS**
https://dauphine.psl.eu/recherche/cvtheque/poisat-julien
https://dauphine.psl.eu/recherche/cvtheque/simenhaus-francois

Langue du cours : Anglais

Volume horaire : 30

Description du contenu de l'enseignement :

The first part of the course (5*3 hours) is devoted to the study of convergence of probability measures on general (that is not necessarily R or Rn) metric spaces or, equivalently, to the convergence in law of random variables taking values in general metric spaces. If this study has its own interest it is also useful to prove convergence of sequences of random objects in various random models that appear in probability theory. The main example we have to keep in mind is Donsker theorem that states that the path of a simple random walk on Z converges after proper renormalization to a brownian motion. We will start this course with some properties of probability measures on metric spaces and in particular on C([0, 1]), the space of real continuous function on [0, 1]. We will then study convergence of probability measures, having for aim Prohorov theorem that provides a useful characterization of relative compactness via tightness. Finally we will gather everything to study convergence in law on C([0, 1]) and prove Donsker therorem. If there is still time we will consider other examples of applicatioin. The main reference for this first part of the course is Convergence of probability measures, P. Billingsley (second edition). The second part of the course will deal with the theory of large deviations. This theory is concerned with the exponential decay of large fluctuations in random systems. We will try to focus evenly on establishing rigorours results and on discussing applications. First, we will introduce the basic notions and theorems: the large deviation principle, Kramer theorem for independent variables, as well as GŁrtner-Ellis and Sanovs theorems. Next, we will see some applications of the formalism. The examples are mainly inspired by equilibrium statistical physics and thermodynamics. They include the equivalence of ensembles, the interpretation of thermodynamical potentials as large deviation functionals, and phase transitions in the mean-field Curie-Weiss model. In a third part, we will develop large deviation principles for Markovian dynamical processes. If times allows, we will present some applications of these results in a last part of the course. There is no explicit prerequisite to follow the classes but students should be well acquainted with probability theory.

Numerical methods for deterministic and stochastic problems

ECTS : 6

Enseignants : LAETITIA **LAGUZET**, GUILLAUME **LEGENDRE**, GABRIEL **TURINICI**
https://turinici.com
https://dauphine.psl.eu/recherche/cvtheque/legendre-guillaume
https://turinici.com

Langue du cours : Anglais

Volume horaire : 45

Description du contenu de l'enseignement :

This course is an introduction to methods for the numerical solution of deterministic and stochastic differential equations and numerical aspects of machine learning. It consists of three distincts parts and includes implementations using Python, FreeFEM++ and Keras/Tensorflow.
Part 1: numerical methods for deterministic partial differential equations Part 2: Monte Carlo methods for particle transport Part 3: machine learning and numerical statistics

- finite difference methods
- finite element methods
- spectral methods
- review of numerical methods for ordinary differential equations
- Monte Carlo integration
- convergence and variance reduction
- transport equations starting from probability measures: examples and numerical methods including particle methods
- high-dimensional statistics and machine learning
- stochastic optimization : SGD, Adam, RMSProp, etc.
- neural networks: architecture, generative paradigms (VAE, GANs, "stable diffusion")

Bibliographie, lectures recommandées :

Part 1

- Randall J. LeVeque, "Finite Difference Methods for Ordinary and Partial Differential Equations: Steady-State and Time-dependent Problems", SIAM (2007)
- Alexandre Ern, Jean-Luc Guermond, "Theory and Practice of Finite Elements", Springer (2004)
- Jie Shen, Tao Tang, Li-Lian Wang, "Spectral Methods. Algorithms, Analysis and Applications", Springer (2011)

Part 2

- C. Graham, D. Talay, "Stochastic Simulation and Monte Carlo Methods", Springer (2013)
- B. Lapeyre, E. Pardoux, R. Sentis, "Introduction to Monte-Carlo Methods for Transport and Diffusion Equations", OUP Oxford (2003)

Part 3

- Ian Goodfellow, Yoshua Bengio, Aaron Courville, "Deep Learning", The MIT Press (2016)
- Alain Berlinet, Christine Thomas-Agnan "Reproducing Kernel Hilbert Spaces in Probability and Statistics", Springer (2011)

See also G. Turinici's web site.

Stochastic calculus

ECTS : 6

Enseignants : CLEMENT **COSCO**, MARC **HOFFMANN**
https://dauphine.psl.eu/recherche/cvtheque/cosco-clement
https://dauphine.psl.eu/recherche/cvtheque/hoffmann-marc

Langue du cours : Anglais

Volume horaire : 48

Description du contenu de l'enseignement :

The first part of the course presents stochastic calculus for continuous semi-martingales. The second part of the course is devoted to Brownian stochastic differential equations and their links with partial differential equations. This course is naturally followed by the course "Jump processes".

- Probability basics
- Stochastic processes
- Brownian motion, Continuous semi-martingales, Stochastic integral, Itô's formula for semi-martingales and Girsanov's theorem Stochastic differential equations
- Diffusion processes Feynman-Kac formula and link with the heat equation Probabilistic representation of the Dirichlet problem

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Cours spécialisés

An introduction to Hyperbolic Systems of Conservation Laws

ECTS : 6

Enseignant responsable : OLIVIER **GLASS** (https://dauphine.psl.eu/recherche/cvtheque/glass-olivier)

Langue du cours : Anglais

Volume horaire : 24

Description du contenu de l'enseignement :

Hyperbolic conservation laws are a class of nonlinear partial differential equations that reflect standard conservation laws of physics (such as conservation of mass, momentum, and energy) and contain many classical models, such as Euler's equations for compressible flows, as well as more modern models for traffic flows, supply chains, etc.

One of the main aspects of these systems is that, regardless of the regularity of the initial data, their solutions generally develop discontinuities in finite time (this mechanism is known as shock formation). Thus, one should consider discontinuous solutions (in the sense of distributions). However, it has been known since Riemann that uniqueness is lost in this context. This motivates the introduction of the concept of entropy solutions: weak solutions fulfilling additional conditions (connected to the second law of thermodynamics in the case of gas dynamics), aimed at recovering uniqueness.

The theory of entropy solutions is now well developed when the space dimension is 1 (but even this case leaves many open questions!) and solutions are of bounded variation. I will mainly focus on this case.

Compétences à acquérir :

Basic theory of 1D hyperbolic systems

Bayesian statistics

ECTS : 6

Enseignant responsable : JUDITH **ROUSSEAU** (https://dauphine.psl.eu/recherche/cvtheque/rousseau-judith)

Langue du cours : Anglais

Volume horaire : 24

Description du contenu de l'enseignement :

The course will cover different aspects of Bayesian statistics with an emphasis on the theoretical properties of Bayesian methods. The course starts with an introduction Bayesian decision theory from point estimation, to credible regions, testing and model selection and some notion on Bayesian predictive inference. The second part will cover the most important results on Bayesian asymptotics.

Part I. Bayesian decision theory : an Introduction

- Prior / Posterior , risks and Bayesian estimators.
- Credible regions.
- Model selection and tests.

Part II: Bayesian asymptotics; in this part, both well and mis-speci?ed models will be considered.

- Asymptotic posterior distribution: in this part we will study asymptotic normality of the posterior, the penalization induced by the prior and the Bernstein von - Mises theorem. Regular and nonregular models will be treated.

- marginal likelihood and consistency of Bayes factors/model selection approaches.
- Empirical Bayes methods. This part will review some results on the asymptotic posterior distribution for parametric empirical Bayes methods.
- Bayesian bootstrap.
- Posterior consistency and posterior convergence rates. This part will ?rst cover the case of statistical loss functions using the theory introduced by L. Schwartz and developed by Ghosal and Van der Vaart

Compétences à acquérir :

Understanding of Bayesian inference and Bayesian decision theory. Understanding and being able to manipulate the asymptotic theory in Bayesian inference : main tools and what they mean.

Pré-requis obligatoires

probability and basics of statistical inference

Pré-requis recommandés

Probability; basics in statistical inference

Mode de contrôle des connaissances :

examen sur table

Bibliographie, lectures recommandées :

- C. P. Robert (2021). The Bayesian Choice.
- S. Ghosal and A. van der Vaart (2017): Fundamentals of Bayesian Nonparametrics.
- A. van der Vaart (1998): Asymptotic Statistics.

Continuous optimization

ECTS : 6

Enseignant responsable : ANTONIN **CHAMBOLLE** (https://dauphine.psl.eu/recherche/cvtheque/chambolle-antonin)

Langue du cours : Anglais

Volume horaire : 24

Description du contenu de l'enseignement :

This course will cover the bases of continuous, mostly convex optimization. Optimization is an important branch of applied industrial mathematics. The course will mostly focus on the recent development of optimization for large scale problems such as in data science and machine learning. A first part will be devoted to setting the theoretical grounds of convex optimization (convex analysis, duality, optimality conditions, non-smooth analysis, iterative algorithms). Then, we will focus on the improvement of basic first order methods (gradient descent), introducing operator splitting, acceleration techniques, non-linear ("mirror") descent methods and (elementary) stochastic algorithms.

Course of PDE and applications

ECTS : 6

Enseignant responsable : PIERRE-LOUIS **LIONS** (https://dauphine.psl.eu/recherche/cvtheque/pierre-louis-lions)

Langue du cours : Anglais

Volume horaire : 18

Determinantal processes, random matrices and hyperuniformity

ECTS : 6

Enseignant responsable : RAPHAEL **LACHIEZE-REY**

Langue du cours : Anglais

Volume horaire : 24

Differential geometry and gauge theory

ECTS : 6

Langue du cours : Anglais

Volume horaire : 24

Dimension reduction and manifold learning

ECTS : 6

Langue du cours : Anglais

Volume horaire : 24

Dynamics of semi-linear wave equation

ECTS : 6

Langue du cours : Anglais

Volume horaire : 28

Description du contenu de l'enseignement :

The aim of this course is to present recent developments concerning the dynamics of non-linear wave equations. In the first part of the course, I will present some classical properties of linear wave equations (cf. [3, Chapter 5]): representation of solutions, finite speed of propagation, asymptotic behavior, dispersion and Strichartz inequalities [7, 5]. The second part of the course concerns semi-linear wave equations. After a presentation of the basic properties of these equations (local existence and uniqueness of solutions, conservation laws, transformations cf. e.g. [5, 6]), I'll give several examples of dynamics: scattering to a linear solution, self-similar behavior and solitary waves. I will also give results on the classification of the dynamics for the energy critical wave equation following [2, 4], and some elements of proofs, including the profile decomposition introduced by Bahouri and Gérard [1]. The prerequisites are the basics of classical real and harmonic analysis. This course can be seen as a continuation of the fundamental courses Introduction to Nonlinear Partial Differential Equations and Introduction to Evolutionary Partial Differential Equations, but can also be taken independently of these two courses. This course will be taught at ENS.

Bibliographie, lectures recommandées :

1. Bahouri, H., and Gérard, P. High frequency approximation of solutions to critical nonlinear wave equations. Amer. J. Math. 121, 1 (1999), 131-175.
2. Duyckaerts, T., Kenig, C., and Merle, F. Classification of radial solutions of the focusing, energy-critical wave equation. Camb. J. Math. 1, 1 (2013), 75-144.
3. Folland, G. B. Introduction to partial differential equations., 2nd ed. ed. Princeton, NJ: Princeton University Press, 1995.
4. Kenig, C. E. Lectures on the energy critical nonlinear wave equation, vol. 122 of CBMS Reg. Conf. Ser. Math. Providence, RI: American Mathematical Society (AMS), 2015.
5. Sogge, C. D. Lectures on nonlinear wave equations. Monographs in Analysis, II. International Press, Boston, MA, 1995.
6. Strauss, W. A. Nonlinear wave equations, vol. 73 of CBMS Regional Conference Series in Mathematics. Published for the Conference Board of the Mathematical Sciences, Washington, DC, 1989.
7. Tao, T. Nonlinear dispersive equations, vol. 106 of CBMS Regional Conference Series in Mathematics. Published for the Conference Board of the Mathematical Sciences, Washington, DC, 2006. Local and global analysis.

Dynamics of gravitational systems with a large number of particles

ECTS : 6

Langue du cours : Anglais

Volume horaire : 21

Description du contenu de l'enseignement :

Reminder on differential equations

- Reminder on Hamiltonian systems
- Short reminder on measure and integration theories
- Elements on distributions
- Application to the Vlasov equation
- The Vlasov-Poisson system
- The BBGKY hierarchy, the hypothesis of molecular chaos
- The particular case of a cluster with spherical symmetry, an explicit solution

This course is taught at Observatoire de Paris.

Bibliographie, lectures recommandées :

- Binney - Tremaine : Galactic Dynamics
- Cours polycopiés de F. Golse

Entropy methods, functional inequalities and applications

ECTS : 6

Enseignants : EMERIC **BOUIN**, JEAN **DOLBEAULT**, Amic **FROUVELLE**
https://dauphine.psl.eu/recherche/cvtheque/bouin-emeric
https://dauphine.psl.eu/recherche/cvtheque/dolbeault-jean

Langue du cours : Anglais

Volume horaire : 24

Description du contenu de l'enseignement :

Various functional inequalities are classically seen from a variational point of view in nonlinear analysis. They also have important consequences for evolution problems. For instance, entropy estimates are standard tools for relating rates of convergence towards asymptotic regimes in time-dependent equations with optimal constants of various functional inequalities. This point of view applies to linear diffusionsand will be illustrated by some results on the Fokker-Planck equation based on the "carré du champ" method introduced by D. Bakry and M. Emery. In the recent years,the method has been extended from linear to nonlinear diffusions. This aspect will be illustrated by results on Gagliardo-Nirenberg-Sobolev inequalities on the sphere and on the Euclidean space. Even the evolution equations can be used as a tool for the study of detailed properties of optimal functions in inequalities and their refinements. There are also applications to other equations than pure diffusions: hypocoercivity in kinetic equations is one of them. In any case, the notion of entropy has deep roots in statistical mechanics, with applications in various areas of science ranging from mathematical physics to models in biology. A special emphasis will be put during the course on the corresponding models which offer many directions for new research development.

Compétences à acquérir :

Various functional inequalities are classically seen from a variational point of view in nonlinear analysis. They also have important consequences for evolution problems. For instance, entropy estimates are standard tools for relating rates of convergence towards asymptotic regimes in time-dependent equations with optimal constants of various functional inequalities. This point of view applies to linear diffusionsand will be illustrated by some results on the Fokker-Planck equation based on the "carré du champ" method introduced by D. Bakry and M. Emery. In the recent years,the method has been extended from linear to nonlinear diffusions. This aspect will be illustrated by results on Gagliardo-Nirenberg-Sobolev inequalities on the sphere and on the Euclidean space. Even the evolution equations can be used as a tool for the study of detailed properties of optimal functions in inequalities and their refinements. There are also applications to other equations than pure diffusions: hypocoercivity in kinetic equations is one of them. In any case, the notion of entropy has deep roots in statistical mechanics, with applications in various areas of science ranging from mathematical physics to models in biology. A special emphasis will be put during the course on the corresponding models which offer many directions for new research development.

Gravitation classique et Mécanique Hamiltonienne

ECTS : 6

Langue du cours : Français

Volume horaire : 30

Description du contenu de l'enseignement :

This course is taught in French at Observatoire de Paris. La mécanique céleste est plus vivante que jamais. Après un renouveau résultant de la conquête spatiale et de la nécessité des calculs des trajectoires des engins spatiaux, un deuxième souffle est apparu avec l'étude des phénomènes chaotiques. Cette dynamique complexe permet des variations importantes des orbites des corps célestes, avec des conséquences physiques importantes qu'il faut prendre en compte dans la formation et l'évolution du système solaire. Avec la découverte des planètes extra solaires, la mécanique céleste prend un nouvel essor, car des configurations qui pouvaient paraître académiques auparavant s'observent maintenant, tellement la diversité des systèmes observés est grande. La mécanique céleste apparaît aussi comme un élément essentiel permettant la découverte et la caractérisation des systèmes planétaires qui ne sont le plus souvent observés que de manière indirecte. Le cours a pour but de fournir les outils de base qui permettront de mieux comprendre les interactions dynamiques dans les systèmes gravitationnels, avec un accent sur les systèmes planétaires, et en particulier les systèmes planétaires extra solaires. Le cours vise aussi à présenter les outils les plus efficaces pour la mise en forme analytique et numérique des problèmes généraux des systèmes dynamiques conservatifs. Plan:

- Le problème des deux corps. Aperçu de quelques intégrales premières, réduction du nombre de degrés de liberté, trajectoire, évolution temporelle. Développements classiques du problème des deux corps
- Introduction à la mécanique analytique. Principe de moindre action, Lagrangien, Hamiltonien
- Équations canoniques. Crochets de Poisson, intégrales premières, transformations canoniques
- Propriétés des systèmes Hamiltoniens. Systèmes intégrables. Flot d'un système Hamiltonien
- Intégrateurs numériques symplectiques
- Systèmes proches d'intégrable. Perturbations. Série de Lie
- Développement du potentiel en polynômes de Legendre
- Évolution à long terme d'un système planétaire hiérarchique, mécanisme de Lidov- Kozai. Application aux exoplanètes
- Mouvements chaotiques
- Exposants de Lyapounov
- Analyse en fréquence.

High-dimensional statistics

ECTS : 6

Enseignant responsable : VINCENT **RIVOIRARD** (https://www.ceremade.dauphine.fr/~rivoirar/)

Langue du cours : Anglais

Volume horaire : 24

Integrable probability and the KPZ universality class

ECTS : 6

Enseignant responsable : GUILLAUME **BARRAQUAND**

Langue du cours : Anglais

Volume horaire : 24

Description du contenu de l'enseignement :

Integrable probability is a relatively new subfield of probability that concerns the study of exactly solvable probabilistic models and their underlying algebraic structures. Most of these so-called integrable models come from statistical physics. They serve as toy models to discover the asymptotic behavior common to large classes of models, called universality classes. The methods used in integrable probability often come from other areas of mathematics (such as representation theory or algebraic combinatorics) and from theoretical physics. In the last twenty years, integrable probability has been particularly fruitful for studying the Kardar-Parisi-Zhang universality class (named after the three physicists who pioneered the domain in the 1980s). This class gathers interface growth models describing a wide variety of physical phenomena, whose asymptotic behavior is surprisingly related to the theory of random matrices.
After a general introduction to the field, we will study in details a central tool in the field, the Schur processes, which allow to study some of the most emblematic integrable models, such as last passage percolation. Then we will consider generalizations and eventually arrive at the the exact calculation of the law of a solution of the Kardar-Parisi-Zhang equation. Along the way, we will take a few detours through connected areas: random matrices, Robinson-Schensted-

Knuth correspondence, interacting particle systems, Yang-Baxter equation and the six-vertex model, random walks in a random environment.
The course will be taught at ENS.

Compétences à acquérir :

Schur functions and measures, elements of random matrix theory, determinantal point processes, asymptotic analysis of Fredholm determinants, elements of stochastic PDEs,

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Introduction to control theory

ECTS : 6

Enseignant responsable : DELPHINE **BRESCH-PIETRI**

Langue du cours : Anglais

Volume horaire : 28

Description du contenu de l'enseignement :

This course focuses on an introduction to systems and control theory. It concerns the study of a dynamical system affected by an input signal which we aim at designing to modify the system behavior. It will focus on nonlinear Ordinary Differential Equations (ODEs), but will also include an introduction to the control of Partial Differential Equations. We will start by reviewing stability notions of nonlinear ODEs (Lyapunov theorems, sufficient and necessary stability conditions, spectral criteria for linear systems, Input-to-State Stability,…). Then, we will study the concepts of controllability/observability of dynamical systems and move to stabilization of equilibrium points, with the presentation of a few control design methodologies (backstepping, forwarding, optimal control, Lie Bracket methods...). The class will be concluded by a few session on the extension of these concepts to infinite-dimensional linear control systems, namely, Partial Differential Equations. Examples will include in-domain and/or boundary control of the heat equation and the wave equation. The course will be taught at École des Mines.

Introduction to generative modeling with flows and diffusions (ENS)

ECTS : 6

Langue du cours : Anglais

Volume horaire : 24

Introduction to statistical mechanics

ECTS : 6

Enseignants : ELEANOR **ARCHER**, BEATRICE **TAUPINART DE TILIERE**
https://dauphine.psl.eu/recherche/cvtheque/archer-eleanor
https://dauphine.psl.eu/recherche/cvtheque/de-tiliere-beatrice

Langue du cours : Anglais

Volume horaire : 24

Description du contenu de l'enseignement :

The aim of statistical mechanics is to understand the macroscopic behavior of a physical system using a probabilistic model containing information about its microscopic interactions. The goal of this course is to give an introduction to this broad subject, which lies at the intersection of many areas of mathematics: probability, graph theory, combinatorics, algebraic geometry…
In the course we will study several important models from the theory of equilibrium statistical mechanics. In particular we will study the phase diagram of the Ising model (ferromagnetism) and of dimer models (crystal surfaces). We will also study uniform spanning trees, including their links to electrical networks, sampling algorithms and connectivity properties.

Compétences à acquérir :

- Formulate Boltzmann measures to define various examples of statistical mechanics models (Ising model, dimers and uniform spanning trees).

- Understand how microscopic interactions on graphs or lattices lead to phase transitions and large-scale phenomena in these models.
- Use combinatorial and graph-theoretic techniques to count configurations, study connectivity, and understand macroscopic structure in these models.

Lie Groups, Lie algebras and represention

ECTS : 6

Langue du cours : Anglais

Volume horaire : 65

Description du contenu de l'enseignement :

The theory of groups and their representations is a central topic which studies symmetries in various contexts occurring in pure or applied mathematics as well as in other sciences, most notably in physics. Lie theory (i.e. the study of Lie groups and Lie algebras) has played an important role in mathematic ever since its introduction by the Norwegian mathematician Sophus Lie in the 19th century. It has had a profound impact in physics as well. The aim of this course is to provide an introduction, from the mathematical perspective, of the classical concepts and techniques of Lie theory. The course will in particular deal with Lie groups, Lie algebras (of finite dimension) and their representations, and include the study of numerous examples. This course will be taught at ENS. Link to the course

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Mean field games theory

ECTS : 6

Enseignant responsable : CHARLES **BERTUCCI** (https://charles-bertucci.github.io/)

Langue du cours : Anglais

Volume horaire : 24

Description du contenu de l'enseignement :

The course will be divided into three parts. In the first one, we will study various concepts of analysis on the spaces of measures (differentiability, different metrics, convexity, monotonicity, optimality conditions, ...). In the second part, we will apply those concepts to the study of mean field optimal control (MFC), which are optimization problems in which the system to control is made of a large number of very small parts. The third part of the course will be devoted to the study of mean field games (MFG), which are games in which a large number of very small players are facing each other. The course will be at the meeting point of analysis, measure theory and stochastic calculus.

Compétences à acquérir :

Pré-requis recommandés

The course on Stochastic Control (1rst semester) is a necessary prerequisite.

Mode de contrôle des connaissances :

Written exam

Random geometric models

ECTS : 6

Langue du cours : Anglais

Volume horaire : 26

Description du contenu de l'enseignement :

We will discuss among the others: the Erdos-Reny graph, the configuration model, unimodular random graphs, Poisson point processes, hard core point processes, continuum percolation, Boolean model and coverage process, and stationary Voronoi percolation. Our main goal will be to discuss the similarities and the fundamental relationships between the different models.

Compétences à acquérir :

This course provides a quick access to some popular models in the theory of random graphs, point processes and random sets. These models are widely used for the mathematical analysis of networks that arise in different applications: communication and social networks, transportation, biology...

Random walks and random media

ECTS : 6

Enseignants : ANTOINE **JEGO**, FRANCOIS **SIMENHAUS**
https://dauphine.psl.eu/recherche/cvtheque/jego-antoine
https://dauphine.psl.eu/recherche/cvtheque/simenhaus-francois

Langue du cours : Anglais

Volume horaire : 30

Description du contenu de l'enseignement :

- **Random walks in random environment** are random processes obtained after launching a Markovian walker on **Z**d equipped with a random field of transition probabilities. We will review classical results (recurrence / transience, LLN, Sinai regime, Kesten Kozlov Spitzer regime) in dimension d=1 where the behaviour of the walk is well understood but also study the difficult multidimensional case d>=2 where even simple questions (as LLN) remains open.
- **Potential theory and electrical networks**: the analogy with electrical networks gives a physical insight as well as a robust method for proving recurrence or transience of reversible random walks on the Euclidean lattice or more general graphs.
- **Random interlacement**, introduced by Sznitman in the early 2010, may be seen as a « soup » of random walk paths. It plays an decisive role both as a limit object for many random walk models and also as a tractable long range correlated random field.

Compétences à acquérir :

Understand and apply the fundamental definitions, results and proofs of the course.

Rough paths and SDE

ECTS : 6

Langue du cours : Anglais

Volume horaire : 24

Spectral theory and variational methods

ECTS : 6

Enseignants : Eric **CANCES**, MATHIEU **LEWIN**
https://dauphine.psl.eu/recherche/cvtheque/lewin-mathieu

Langue du cours : Anglais

Volume horaire : 21

Stochastic control

ECTS : 6

Enseignant responsable : PHILIPPE **BERGAULT** (https://dauphine.psl.eu/recherche/cvtheque/bergault-philippe)

Langue du cours : Anglais

Volume horaire : 24

Description du contenu de l'enseignement :

PDEs and stochastic control problems naturally arise in risk control, option pricing, calibration, portfolio management, optimal book liquidation, etc. The aim of this course is to study the associated techniques, in particular to present the notion of viscosity solutions for PDEs.

- Relationship between conditional expectations and parabolic linear PDEs
- Formulation of standard stochastic control problems: dynamic programming principle.
- Hamilton-Jacobi-Bellman equation
- Verification approach Viscosity solutions (definitions, existence, comparison)
- Application to portfolio management, optimal shutdown and switching problems

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Variational and geodesic methods for Image Analysis

ECTS : 6

Enseignant responsable : LAURENT **COHEN** (https://www.ceremade.dauphine.fr/~cohen/)

Langue du cours : Anglais

Volume horaire : 24

Description du contenu de l'enseignement :

This course, after giving a short introduction to digital image processing, will present an overview of variational methods for Image segmentation. This will include deformable models, known as active contours, solved using finite differences, finite elements, level sets method, fast marching method. A large part of the course will be devoted to geodesic methods, where a contour is found as a shortest path between two points according to a relevant metric. This can be solved efficiently by fast marching methods for numerical solution of the Eikonal equation. We will present cases with metrics of different types (isotropic, anisotropic, Finsler) in different spaces. All the methods will be illustrated by various concrete applications, like in biomedical image applications.

Compétences à acquérir :

Basic knowledge about partial differential equations is better. The course will give mathematical formulation of various problem followed by numerical methods to solve these problem and algorithmic considerations. Concrete applications of these problems will be presented.

Variational problems and optimal transport

ECTS : 6

Enseignant responsable : GUILLAUME **CARLIER** (https://dauphine.psl.eu/recherche/cvtheque/carlier-guillaume)

Langue du cours : Anglais

Volume horaire : 24

Description du contenu de l'enseignement :

Chapter 1: Convexity in the calculus of variations

- separation theorems, Legendre transforms, subdifferentiability
- convex duality by a general perturbation argument, special cases (Fenchel-Rockafellar, linear programming, zero sum games, Lagrangian duality)
- calculus of variations: the role of convexity, relaxation, Euler-Lagrange equations

Chapter 2: The optimal transport problem of Monge and Kantorovich

- The formulations of Monge and Kantorovich, examples and special cases (dimension one, the assignment problem, Birkhoff theorem), Kantorovich as a relaxation of Monge
- Kantorovich duality
- Twisted costs, existence of Monge solutions, Brenier's theorem, Monge-Ampère equation, OT proof of the isoperimetric inequality
- Introduction to entropic optimal transport

Chapter 3: Dynamic optimal transport, Wasserstein spaces, gradient flows

- The distance cost case and its connection with minimal flows
- Wasserstein spaces
- Benamou-Brenier formula and geodesics, displacement convexity

- gradient flows, a starter: the Fokker-Planck equation, general theory for lambda-convex functionals

Compétences à acquérir :

Maitrise des outils d'analyse convexe pour le calcul des variations et des éléments fondamentaux de la théorie du transport optimal

Pré-requis recommandés

Bonnes bases d'analyse fonctionnelle et de théorie de la mesure

Mode de contrôle des connaissances :

Examen écrit ou oral

SEMESTRE 4 - 18 ECTS

Bloc mémoire

Mémoire de recherche

ECTS : 18

Langue du cours : Français
