

Dynamics of semi-linear wave equation

ECTS : 6

Volume horaire : 28

Description du contenu de l'enseignement :

The aim of this course is to present recent developments concerning the dynamics of non-linear wave equations. In the first part of the course, I will present some classical properties of linear wave equations (cf. [3, Chapter 5]): representation of solutions, finite speed of propagation, asymptotic behavior, dispersion and Strichartz inequalities [7, 5]. The second part of the course concerns semi-linear wave equations. After a presentation of the basic properties of these equations (local existence and uniqueness of solutions, conservation laws, transformations cf. e.g. [5, 6]), I'll give several examples of dynamics: scattering to a linear solution, self-similar behavior and solitary waves. I will also give results on the classification of the dynamics for the energy critical wave equation following [2, 4], and some elements of proofs, including the profile decomposition introduced by Bahouri and Gérard [1]. The prerequisites are the basics of classical real and harmonic analysis. This course can be seen as a continuation of the fundamental courses Introduction to Nonlinear Partial Differential Equations and Introduction to Evolutionary Partial Differential Equations, but can also be taken independently of these two courses. This course will be taught at ENS.

Bibliographie, lectures recommandées :

1. Bahouri, H., and Gérard, P. High frequency approximation of solutions to critical nonlinear wave equations. Amer. J. Math. 121, 1 (1999), 131-175.
2. Duyckaerts, T., Kenig, C., and Merle, F. Classification of radial solutions of the focusing, energy-critical wave equation. Camb. J. Math. 1, 1 (2013), 75-144.
3. Folland, G. B. Introduction to partial differential equations., 2nd ed. ed. Princeton, NJ: Princeton University Press, 1995.
4. Kenig, C. E. Lectures on the energy critical nonlinear wave equation, vol. 122 of CBMS Reg. Conf. Ser. Math. Providence, RI: American Mathematical Society (AMS), 2015.
5. Sogge, C. D. Lectures on nonlinear wave equations. Monographs in Analysis, II. International Press, Boston, MA, 1995.
6. Strauss, W. A. Nonlinear wave equations, vol. 73 of CBMS Regional Conference Series in Mathematics. Published for the Conference Board of the Mathematical Sciences, Washington, DC, 1989.
7. Tao, T. Nonlinear dispersive equations, vol. 106 of CBMS Regional Conference Series in Mathematics. Published for the Conference Board of the Mathematical Sciences, Washington, DC, 2006. Local and global analysis.

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