An introduction to Hyperbolic Systems of Conservation Laws

ECTS : 6

Volume horaire : 24

Description du contenu de l'enseignement :

Hyperbolic conservation laws are a class of nonlinear partial differential equations that reflect standard conservation laws of physics (such as conservation of mass, momentum, and energy) and contain many classical models, such as Euler's equations for compressible flows, as well as more modern models for traffic flows, supply chains, etc.

One of the main aspects of these systems is that, regardless of the regularity of the initial data, their solutions generally develop discontinuities in finite time (this mechanism is known as shock formation). Thus, one should consider discontinuous solutions (in the sense of distributions). However, it has been known since Riemann that uniqueness is lost in this context. This motivates the introduction of the concept of entropy solutions: weak solutions fulfilling additional conditions (connected to the second law of thermodynamics in the case of gas dynamics), aimed at recovering uniqueness.

The theory of entropy solutions is now well developed when the space dimension is 1 (but even this case leaves many open questions!) and solutions are of bounded variation. I will mainly focus on this case.

Compétence à acquérir :

Basic theory of 1D hyperbolic systems

Document susceptible de mise à jour - 04/04/2026
Université Paris Dauphine - PSL - Place du Maréchal de Lattre de Tassigny - 75775 PARIS Cedex 16