Apprentissage topologique

ECTS : 3

Volume horaire : 24

Description du contenu de l'enseignement :

COURSE CONTENT

1. Topological Foundations

- **From distances to topology**: Metric spaces, open balls, neighborhoods, and the transition to abstract topological spaces.
- **Invariants and deformations**: Connectivity, holes, homotopy, and why topology is more fundamental than geometry for learning.
- **Discrete and continuous structures**: Graphs, hypergraphs, simplicial complexes, CW-complexes, and their geometric realizations.

2. Mathematical Tools for Topological Analysis

- **Combinatorial topology**: Simplices, complexes, cochains, and associated matrices (incidence, adjacency, Laplacian).
- **Algebraic topology**: Homology, cohomology, and applications in Topological Data Analysis (TDA).

3. Deep Learning on Topological Structures

- **Message passing**: Convolutions, attention, and diffusion on graphs and higher-order structures.
- **Spectral representations**: Normalized Laplacian, Fourier analysis on graphs, and implications for learning.
- **Limitations and extensions**: Oversmoothing, heterophily, sheaf theory, and Ricci curvature for more robust models.

4. Applications and Implementation

- **Graphs and hypergraphs**: Social networks, knowledge graph embeddings (TransE), community detection.
- **Simplicial complexes**: 3D mesh processing, molecular classification, and topological signal processing.
- **Practical tools**: Hands-on experience with libraries like TopoNetX to prototype topological models.

5. Critical Review and Perspectives

- **State of the art**: GCN, GAT, Simplicial Neural Networks, and foundational graph models.
- **Open challenges**: Topological identification capacity, spatio-temporal structures, and integration with generative AI.

Compétence à acquérir :

- **Understand and apply** core topological concepts (metric spaces, topological spaces, topological invariants) to data science and machine learning problems.
- **Model and analyze** discrete (graphs, hypergraphs, simplicial complexes) and continuous (manifolds, abstract topological spaces) structures within a unified framework.
- **Design and interpret** topological data representations (embeddings, Laplacian matrices, homology) for deep learning tasks.
- **Implement** message-passing algorithms (GNNs, attention mechanisms, diffusion) and spectral analysis on topological structures.
- **Evaluate** the limitations of existing models (oversmoothing, heterophily) and propose solutions inspired by algebraic or differential topology (sheaf theory, Ricci curvature).
- **Apply** these tools to real-world problems: social networks, 3D mesh processing, molecular classification, dimensionality reduction, and structured data generation.
- **Engage with** recent literature in topological learning, including advances in Topological Data Analysis (TDA), manifold learning, and topological neural networks.